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Article · January 2020

Research Article

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Abstract

Several construction methods have been introduced to build the elements of BIBDs' for specific parameters, with different techniques suggested for testing their existence, still no general technique to determine the efficiencies of these designs has been realized. In this study the efficiencies and relative efficiencies of Sum constructed automorphic symmetric balanced incomplete block designs with respect to parent designs has been presented. The process involved the classical analysis of variance applied to a data set. Computed relative efficiency of the design λ_3 with respect to parent design λ_1 has been found to be 1.216.

Keywords: Relative efficiency; Symmetric Balanced Incomplete Block Design; Automorphic Symmetric Balanced Incomplete Block Design.

Introduction

The recent application of BIBD's are inclined to meet the ever rising and emerging statistical needs [1]. Although a large number of block designs are available in literature the greatest challenge is the overall efficiency of the design to be used because of the fact that efficiency of BIBD in general has received little attention since the inception of design theory [2]. Even though there exist some situations where there are more sources of variation that cannot be controlled by ordinary blocking, efficiency of any design is a vital component as illustrated by [3] on remarks of BIBD's.

A study by [4] on comparative analysis of SBIBD and Asymmetric BIBD reveled the efficiency of such designs. Several other authors have discussed various properties of the BIBD, taking different dimensions in the approach of construction, development and practical application [5]. However, little attention and emphasis has been laid on the efficiency and relative efficiency of AUSBIBD [6]. A BIBD would be best preferred if it has a high precision value irrespective of whether it contains repeated blocks or not. Indeed, the statistical optimality of

BIBD is not affected by the presence of any number of repeated blocks [7].

A study by [8] on SBIBD (v, k, λ) that admits flag-transitive and point-primitive automorphism groups which are highly associating to special two dimensional projective groups showed that for a BIBD the canonical efficiency factor (proportion of the information) within the blocks is $e_2 = \frac{vk}{r\lambda}$ and the canonical efficiency factor between the blocks is $e_1 = 1 - e_2$. These proportions are called the canonical efficiency factors [9]. For a particular randomized term (treatments) the canonical efficiency factors e_1 and e_2 are always values ranging between zero and one. A class of efficiency BIBD were constructed by [10].

The method of incidence matrices led to a high efficient BIBD. While studying on the performance of a series of NBBD, [11] showed that BIBD with moving average correlation structure are more efficient. The current study adds on to the gap of knowledge by providing insights on both the efficiency and relative efficiency of sum constructed AUSBIBD.

The related literature on efficiency and relative Kumarland [6], studied the performance

of a series of Complete and Incomplete NNBD for auto regressive, moving average and nearest neighbor error correlation structure when generalized least squares estimation is used. The study compared the efficiency of moving average, auto regressive and nearest neighbor correlation structures. The main observation of this study was that the efficiency for nearest neighbor correlation structure effect is high, in case of complete block designs. In case of BIBD's moving average correlation structure turned out to be more efficient as compared to others models in the interval 0.1 to 0.4. They concluded that when block sizes are large and nearest neighboring plots are highly correlated, generalized least squares for estimation of direct and nearest neighbor effects can be used.

Rajarathinam, Mahalakshmi and Ghosh [10], presented two new methods for the construction of efficiency BIBD with repeated blocks by using different types of BIBD. Their first method presented the construction of efficiency BIBD using the incidence matrices of two BIBD of the series $v_1 = b_1 = s$, $r_1 = k_1 = s-1$, $\lambda_1 = s-2$ and $v_2 = b_2 = s-1$, $r_2 = k_2 = s-2$, $\lambda_2 = s-3$. Their second method-2 discussed the construction of Efficiency Balanced Block Design based on BIBD whose series is; v, $b = vC_2$, $r = v-1$, $\lambda = 1$ and $k = 2$. As an illustration the study provided numerical examples which so far are praised for a very high efficiency value of approximately 0.81.

A study on construction of efficiency-balanced design using factorial design by Kumarland [6] calculated the efficiency factor (E) of the efficiency balanced design constructed using the expression efficiency factor (E) $=1-\mu$. The approach employed in this study encompassed the idea of the M-matrix of the efficiency balanced design given by is $1-n\mu$, $\mu =+ MIJr^1$ where μ is the loss of information, I represent the identity matrix whose order is (v x v), J represent the unit vector whose order is (v x 1), r^1 is the row vector of order (1 x v), and n is referring to the total observations. In this current study we have equally computed the efficiency factor for sum constructed SBIBD by using the expression of efficiency factor value involving μ as applied in this research and carrying out some simplifications in the efficiency factor formula.

According to a study by Kelechi [5] on the comparative analysis of symmetric and unsymmetric BIBD, minimization of error in BIBD, SBIBD leads to a minimum error than non-symmetric BIBD whether the treatment is adjusted or not. Also it is better to adjust the treatments in SBIBD and non-symmetric BIBD because the adjustment leads to minimum error. Further he found out that the classical ANOVA method on SBIBD seems easier and more convenient or efficient to handle than the classical ANOVA method on non-symmetric BIBD. Their study has been criticized for failing to show the canonical efficiencies of the two designs. The current study fills the gap of knowledge on construction of AUSBIBD by introducing a sum construction method coupled with an algorithm for AUSBIBD and further provides an in depth analysis of the efficiencies of sum constructed AUSBIBD.

On the same note a study by Otulo, Ojunga, and Otumba [7] on screening new strains of sugarcane using augmented block design showed the efficiency of augmented block design and randomized complete block in determine the best performing clones. The findings of this study revealed that augmented block designs are more efficient than RCBD when subjected to the same treatment analysis.

A study by Sinha, Das, Dey, and Kageyama [11], presented a highly A-efficient BIBD used for comparing sets of test treatments and sets of controls. The construction method described by this study has an advantage of being able to use the vast literature on BIBD to obtain a relatively large numbers of highly A-efficient BIBD. The earlier presented BIBDs' have been considered to require a few number of blocks as compared to the A-efficient optimal designs that exist in the literature.

A computation of an A-efficiency for a design used to make test treatment versus control comparisons by applying the procedure outlined by Chan and Eccleston [3] was used in the construction of an A- efficient optimal design. Further, the study findings advised that when comparing a given number of treatments of any BIBD similar conditions must be maintained so as to provide or make a precise measurement of treatment means. This ensures that the difference among treatment means remain so minimal and

may not result from extraneous factors other than application factors. To attain this, experimental trials for BIBDs' are often grouped to form homogenous blocks with constant conditions maintained in such blocks.

In order to eliminate heterogeneity and improve on the accuracy or efficiency of any BIBD, the current study has introduced a new concept of sum construction of AUSBIBD. In this newly constructed design heterogeneity is reduced to a greater extent than is possible with randomized block designs, Latin square designs and initial SBIBD. As a further development in bridging the gap of knowledge along this line, this study describes a practical application revealing the relative efficiency of sum constructed AUSBIBD in the field of forestry.

Preliminaries

Definition 1.1

Efficiency of a design is a measure of extent of quality of an experimental design or of a hypothesis testing.

Remark 1.2

An efficient design is characterized by small variance or small mean square error indicating that there exists a very small deviation between the true value and the estimate.

Definition 1.3

The relative efficiency of two designs is the ratio of their individual efficiencies.

Remark 1.4

For any outlined procedure the relative efficiency depends squarely on the sample sizes chosen.

Definition 1.5

A bijection α is referred to as an isomorphism if every treatment $y \in Y$ is renamed by $\alpha(y)$ and the collection of blocks in A are transformed into B for any two designs (**Y, A**) and (**X, B**) with $|Y|=|X|$.

Example 1.6

Given two (7,3,2) SBIBD's that is to say (Y, A) and (X, B) whose treatment values are Y = {1,2, 3, 4, 5, 6, 7} and the blocks of the first design A = {123,345,567,127,234,456,167} respectively. By letting the treatment values and blocks in the

second design be X = {a, b, c, d, e, f, g} and B = {abd, dcg, gef, fab, bdc, acge, efa} respectively and Supposing a bijection α is defined by
$\alpha(1)$ = a, $\alpha(3)$ = d, $\alpha(4)$ = c, , $\alpha(2)$ = b, $\alpha(5)$ = g, $\alpha(7)$ = f and $\alpha(6)$ = e
By relabeling the points in Y using the bijection the blocks in A yields to:-
123−abd
345−dcg
567−gef
127−abf
234−bdc
456−cge
167−aef
Therefore, α is called an isomorphism on SBIBD.

Definition 1.7

An automorphism is a symmetry preserving the permutation or bijective function from a block of a design onto itself.

Remark 1.8

In general, a group of permutations on the points of a design that preserves its blocks is called an automorphism group of that design.
Parvathy and Bury [9] provided a theorem on isomorphisms and automorphisms in relation to BIBD, this was possible in a study on theory of block designs. The statement of the theorem on isomorphism and automorphism is cited in the current study as;

Theorem 1.9

Let M and N be incidence matrices of two BIBD's whose parameters are (v, b, r, k, λ). The given BIBDs' are isomorphic if there exists a permutation matrices **P** and **Q** of orders *v x v* and *b x b* respectively, such that **M = PNQ.**

Proof

Let **P** and **R** be matrices where (i, γ(i))th entry and (j, β(j))th entry are 1's and rest elements are all zeros respectively. Further let **R^{T} = Q** with **P** and **Q** being permutation matrices, **PN** becomes a rearrangement of rows in matrix **N** which is corresponding to the bijection action on all points. A Post multiplication by matrix **Q** lead to a rearrangement of columns. In all the actions the structure is preserved as no matrix column elements are not changed. The results of the study by Parvathy and Bury **[9]** showed that α permutations on a set Y may be given as disjoint

cycle representation, where every disjoint cycle taking the form; (y, α (y), α (α (y))….) for all values of y \in Y. The obtained cycles are known to be disjoint with lengths summing to modulus Y. The α permutation order is given by the value of the least common multiple of the cycle lengths. They further showed that a SBIBD automorphism $S_{|X|}$ is formed by all sets of SBIBD automorphisms that are under the set operation composing many permutations. Theorems formulated in this study have been very instrumental in the current study. Most of the proofs of the theorems used in sum construction method in the current study follow from results of the study on introduction to theory of BIBD by Fisher [4].

Research methodology

This section reveals the tools used in data collection and analysis.

Model

The analysis of SBIBD has been made easy by the introduction of software like R-studio where, once the relevant script is written and the output run with specification of the design in question, the results for the performed test are displayed as the output. We choose an additive fixed model for any SBIBD whose parameters are D (v, b, r, k, λ).
The model used is; $\gamma_{ijk} = \mu + t_{ij} + \beta_{ij} + \varepsilon_{ijk}$ where: -
μ is the overall mean
t_{ij} is the ith treatment effect due to the jth block,
β_{ij} is the jth block effect due to the ith treatment and
ε_{ijk} is random effect which is *i.i.d* with a $\boldsymbol{\mu} = 0$ and variance $=\boldsymbol{\sigma^2}$.
The model used, is for a SBIBD which is believed to yield several automorphisms on application of the sum construction method. The analysis of the chosen model leads us to the attainment of least squares means from estimated parameters that results from the least squares fit of the model. Since the least squares means come from the fitted model regardless of the presence and pattern of missing data for the design, this method is preferred. In our case the least squares means for treatments corresponded to the combined intra- and inter-block estimates of the treatment effects as required to be fulfilled for the analysis in R studio.

The ANOVA table of AUSBIBD was then constructed after determining the following; Variation sources, degrees of freedom for treatment effect (v -1), replication (r - 1) and for the error term (r - 1)(v - 1).
In the calculations of the sum of squares we considered the following assumptions for the experimental design
(i). The observations are normally distributed.
(ii). The observations are independently distributed.
(iii). The variance of the error is constant for the F-test to hold.
The total sum of squares has been regarded as the total sum of deviations of individual observations of diameter and breast height of the trees from the mean of all of them considered together. The following computations were considered: -

(i). Total sum of squares $SS_T = \sum Y_{ij}^2 - \frac{Y_{...}^2}{N}$ where $\sum Y_{ij}^2 =$ Sum of squares of deviations of each treatment.
(ii). Sum of squares due to treatments were treated as the sum of squares among the groups with the equation for computations provided as;
$SS_t = \frac{\sum Y_i^2}{v} - \frac{Y_{...}^2}{N}$
(iii). Sum of squares due to blocks were treated as the sum of squares between the groups with equation for computations provided as; $SS_b = \frac{\sum Y_{.j}^2}{b} - \frac{Y_{...}^2}{N}$
(iv). Sum of squares due to residuals or error variations were then computed using subtraction as $SS_E = SS_T - (SS_t + SS_b)$
The organization and presentation of the Analysis of variance table constructed without considering adjustment of both treatments and blocks, following the results from the computation formulae provided above appear in table 1.

Table 1. ANOVA table without considering adjustment of treatments and blocks

SoV	SS	df	MSS	F-value
Treatments	$\frac{\sum Y_i^2}{v} - \frac{Y_{...}^2}{N}$	$v - 1$	$\frac{SST}{v - 1}$	$\frac{MST}{MSE}$
Blocks	$\frac{\sum Y_{.j}^2}{b} - \frac{Y_{...}^2}{N}$	$b - 1$	$\frac{SSB}{b - 1}$	$\frac{MSB}{MSE}$
Error	SS_E (by subtraction)	$vb - v - b + 1$	$\frac{SS_E}{vb - v - b + 1}$	
Total	$\sum Y_{ij}^2 - \frac{Y_{...}^2}{N}$	$vb - 1$		

Sum Construction method

Given two designs that are on the same point set most preferably SBIBD's, sum construction involves systematic addition of a fixed value to the treatments in any block of a given design to form a collection of all the blocks. By fixing some parameters, a new design is obtained through a method of construction known to this study as sum construction. In this work, we have concentrated on the sum construction of AUSBIBD. The theorems below have been fully employed in this work

Theorem 1.10

If (X_1, A_1) and (X_1, A_2) are two SBIBD existing on a set X with parameters (v, k, λ_1) and a (v, k, λ_2) respectively then (v, k, ($\lambda_1 + \lambda_2$)) exists on the same set.

Proof

Let $A = A_1 \cup A_2$ represent the union containing the sets A_1 and A_2 then A is a multi-set of non-empty subsets of X, clearly $|X|$=v, furthermore since every block in A_1 contains k points and we have that every blocks in A_2 also contain k points. Then it follows that A too has k points
If $(x_1, y) \in X$ are chosen such that x≠y, and that the pair (x, y) are members of λ_1 blocks in the set A_1 and the pair (x, y) are members of λ_2 blocks in the set A_2 then the pair (x, y) is impliedly contained in $\lambda_1 + \lambda_2$ blocks in the set A. This is proved true for any arbitrarily selected points x, y ∈ X, therefore, (X, A) become a (v, k, $\lambda_1 + \lambda_2$)-SBIBD.

Corollary 1.11

If a design (X, A) with parameters (v, k, λ) is a BIBD existing on the set X, then for every positive integer **p** ≥ 1 a BIBD (X, A^*) whose parameters are (v, k **p**λ) exist on X

Proof

Let be a positive integer such that p ≥1 and the set A^* be the union of the multisets of A with itself up to **p** times i.e $A^* = A \cup A \cup A \ldots \ldots \cup A$. By theorem 3.1, the design(X, A^*) is a BIBD whose parameters are $(v, k, \lambda + \lambda, \ldots \ldots + \lambda) = (v, k \, p\lambda)$. This implies the sum construction is carried on the multi set union **p** times.

Data set

The data set was obtained from Kenya Forest Research Institute - Maseno branch on breast height and diameters of trees. As per the designs the data set with a single replication was considered to represent design D_1 (v, b, λ_1), this was so because each pair of treatment were only appearing ones hence justifying the case λ_1=1. The set with two replicates represented design D_2 (v, b, λ_2), in which each pair of treatment appeared exactly twice hence justifying the case λ_2 =2. Equally, the set with three replicates represented design D_3 (v, b, λ_3) in which each pair of treatment appeared exactly three times hence justifying the case $\lambda_3 = \lambda_1 + \lambda_2$ =3. The later design was used as the sum constructed design with ($\lambda_1 + \lambda_2$). The three designs were subjected to analysis of variance in a bid reveal which design amongst the three was more efficient than the others.

Efficiency and Relative Efficiency

The following relationship was used to estimate the overall efficiency of any SBIBD

$$\text{Efficiency (E)} = \frac{v}{v-1} \times \frac{k-1}{k}$$

Further, some calculations intended to reveal the relative efficiencies (I) of the AUSBIBD in relation to the parent design has been achieved using; Relative efficiency $(I_{1:3}) = \frac{I_1}{I_3}$

The block intra-section method

Having an existing BIBD whose parameters are D (v, b, r, k, λ), a new BIBD with (b- 1) blocks is formed using the block intra-section method by deleting any of the blocks in the existing BIBD and equally deleting all the treatments of the deleted blocks. The resulting remaining treatments were then re-arranged and numbered as 1, …., v-1. It is always known that whenever block intra-section procedure is applied for a SBIBD whose parameters are D (v, k, λ), the resulting new design has the following parameters v* = v -k, b* = b -1, r* = r, k* = k- 1, λ*= λ this implies that the symmetry property is not maintained.

Block intersection method

A BIBD whose parameters are D (v, b, r, k, λ) can be remodeled by deleting those treatments which were missing in the deleted block and numbering the remaining treatments from

1,….,k. The parameters of resulting design from SBIBD take the form; $v^* = k$, $r^* = r-1$, $b^* = b - 1$, $k^* = \lambda$, $\lambda^* = \lambda -1$, as proposed by Allan **[2]**. Equally when block intersection is used on a SBIBD, the symmetry property of the design is not maintained.

Results and discussions

Analysis of variance for the Sum Constructed AUSBIBD

Two parent designs marked as fit 1 and fit 2 were used in the sum construction of a third design marked as fit 3 when the data set was subjected to an ANOVA test. The following results were obtained

Fit 3

> fit3 = aov (DBH~Tree*Block,data = sum constructed AUSBIBD)
Analysis of variance for fit 3 is given in table 2.
Response: DBH

Table 2. Analysis of variance table for fit 3

S.o.V	D.f	S.S	M.S.S	F-value	Pr(>F)
Tree	1	54.601	54.601	5.0294	0.0265 *
Block	1	1.11	1.110	0.1023	0.7496
Tree: Block	1	2.43	2.427	0.2235	0.7496
Residuals	139	1509.04	10.856		

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Design three reveal that there exist significant variations in the means of the pair of trees at 95% C.I with a p-value of 0.0265.

Fit 2

> fit2=aov(DBH~Tree*Block,data=design2)
Anova for fit 2 is given in table 2.
Response: DBH

Table 3. Anova table for fit 2

S. o .V	D.f	S. S	M. S. S	F-calc.	Pr(>F)
Tree	1	6.42	6.4163	0.3686	0.5453
Block	1	28.80	28.7986	1.6545	0.2016
Tree:Block	1	3.38	3.3756	0.1939	0.6607
Residuals	91	1583.93	17.4058		

Design two reveal that there exist no significant variations in the means of the pair of trees at 95% C.I with all the p-values > 0.05, It is therefore in our case considered ales efficient design

Fit 1 or Design 1

Anova for fit 2 is given in table 2.
Response: DBH

Table 4. Analysis of variance table for fit 1

S.o.V	D.f	S. S	M. S. S	F-calc.	Pr(>F)
Tree	1	34.38	8.595383	3.9578	0.05289 .
Block	1	27.59	27.591	3.1760	0.08163 .
Tree:Block	1	11.33	11.334	1.3047	0.25953
Residuals	44	382.24	8.687		

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Design one reveal that there exist no significant variations in the means of the pair of trees at 95% C.I with all the p-values > 0.05, It is therefore considered a less efficient design. We consider a design to be more powerful or more efficient if in its analysis, more of the variables prove to be significant. This is only possible when the designs under comparisons are subjected to a similar analysis.

It is clear that design 3 which is considered a sum constructed design has more variables being significant hence considered to be a more efficient as compared to design one and design two, where the statistical analysis indicated fewer or no variable as being significant.

Relative Efficiency of AUSBID

We employed the concept of the relative efficiency to formalize the comparison between the sum constructed design and the two initial experimental designs. This is made possible by quantifying the balance between loss of degrees of freedom and the experimental error reduction. The relationship, $I = \frac{1}{\sigma_\varepsilon^2}$ is used in providing the needful information per replication in every design considered in this study.

The relative efficiency of design 1 to design 3 was computed using,

Relative efficiency $(I_{1:3})$ =

$$\frac{I_1}{I_3} = \frac{1}{\sigma_\varepsilon^2} = \frac{(dfMSE_1 +1)(dfMSE_3 +3)(MSE_3)}{(dfMSE_3 +1)(dfMSE_1 +3)(MSE_1)}$$

Since the true experimental error value is not known due uncontrolled variations. We account for this gap of knowledge by introducing a correction factor to the expression provided in equation 4.1, this is done to provide more information per replication.

The modified expression that includes the correction factor in order to obtain a better estimate of the relative efficiency, appear as; $I_{1:3}$

$$= \frac{I_1}{I_3} = \frac{(dfMSE_1 + 1)(dfMSE_3 + 3)\,(MSE_3)}{(dfMSE_3 + 1)(dfMSE_1 + 3)\,(MSE_1)}$$

Upon substituting the actual values, the calculations give the relative efficiency as; $I_{1:3}$

$$= \frac{I_1}{I_3} = \frac{(44+1)(139+3)(10.856)}{(139+1)(44+3)(8.67)} = \frac{69369.84}{57048.6} = 1.216$$

A relative efficiency value of 1.216 imply that the design three considered as the sum constructed AUSBIBID is 21.6 percent more efficient than the parent design and that it provides more information than design one.

The computation was done to compare design three and design two, the value was found to be 1.96 implying that the sum constructed was 9.6 percent more efficient when compared with design two. From the values of Relative efficiencies computed the sum constructed ASBIBD is more efficient

Efficiency of the designs

The efficiencies of the designs were computed using the relationship describe in equation *3.3.1*. Taking the test design with the values of the parameters specified above as v=7, b=7, k= 3, r=3 and λ=2, the efficiency was computed

$$\frac{7-1}{7} \times \frac{3-1}{3} = \frac{12}{21} = 77.78\%$$

A consideration to the set of parameters v=12, b=22, k= 11 r=6, $ and $\lambda=2$ was equally given attention and the result yielded a 76.38%. Other consideration for the set of parameters (v, k, \lambda) = (37, 13, 7) and (32, 9, 2) yielded a 94.87% and 91.75% efficiencies values respectively. Then efficiencies of other BIBDs' computed using the same relationship were summarized in table 5. It is worth noting that the relative efficiency increases with increase in treatment for symmetric BIBDs.

Table 5. Efficiencies of other BIBDs'

Design	Replication treatment ratio	Efficiency %
37, 13, 7	13:37	94.87
32, 9, 2	9:32	91.75
7, 3, 1	3:7	77.78
12, 22, 11, 6, 2	1:2	76.38

Conclusions

It is concluded that the sum constructed AUSBIBD reveal more information per block

than the parent designs involved in their construction. The calculated relative efficiencies were at 1.216 and 1.96 with respect to design one and two respectively. A confirmation that the sum constructed design provides more information per block, hence more efficient. An increase in the number of treatments equally leads to an increase in the efficiency of the design.

Conflicts of interest

Authors declare no conflict of interest.

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