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Mathematical description of a bounded oil reservoir with a horizontal well: early time flow period

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ABSTRACT

Horizontal wells are more productive compared to vertical wells if their performance is optimized. For a completely bounded oil reservoir, immediately the well is put into production, the boundaries of the oil reservoir have no effect on the flow. The pressure distribution thus can be approximated with this into consideration. When the flow reaches either the vertical or the horizontal boundaries of the reservoir, the effect of the boundaries can be factored into the pressure distribution approximation. In this paper we consider the above cases and present a detailed mathematical model that can be used for short time approximation of the pressure distribution for a horizontal well with sealed boundaries. The models are developed using appropriate Green's and source functions. In all the models developed the effect of the oil reservoir boundaries as well as the oil reservoir parameters determine the flow period experienced. In particular, the effective permeability relative to horizontal anisotropic permeability, the width and length of the reservoir influence the pressure response. The models developed can be used to approximate and analyze the pressure distribution for horizontal wells during a short time of production. The models presented show that the dimensionless pressure distribution is affected by the oil reservoir geometry and the respective directional permeabilities.

Key Words: Horizontal well, pressure distribution, bounded oil reservoir, permeability, short time approximation.

INTRODUCTION

The transient flow behavior for horizontal wells is not as direct as it is for vertical wells due to the many boundaries that can affect the flow as compared to vertical wells. A well that is turned horizontally is considered to be a horizontal well otherwise vertical. A bounded oil reservoir has sealed outer boundaries which is different from an infinite acting reservoir. This makes it impossible to have a flow across its boundaries A horizontal well in an oil reservoir with sealed boundaries considers a case where there is no flow in the x, y and z boundaries. The permeability in the three directions is not necessarily constant as is assumed in many models. The aim of this study is to develop a mathematical model that accounts for anisotropic permeability for a horizontal well in a completely bounded oil reservoir as well as account for the effects of the boundaries.

There has been a lot of effort as many authors try to develop better mathematical models for pressure distribution for horizontal wells. Carslaw and Jaeger (1959) came up with results that could be applied in transient flow which has continued to influence fluid flow modelling. Advances in well test analysis have continued to be based on the monograph developed by Mathews and Russell (1967). Gringarten and Ramey (1987) detailed the use source and Green's functions in solving unsteady flows. Lee (1982) discussed the use of source and Green's functions to model pressure in a bounded oil reservoir during early time. Over the years different authors used these source functions to model pressure (Daviau et al., 1985; Clonts & Ramey;

1986; Ozkan et al., 1987; Odeh & Babu, 1987, 1989; Carvalho & Rosa, 1988). Adewole (2009, 2010) used source and Green's functions to develop solutions for pressure distribution for bounded oil reservoirs. Al Rbeawi and Tiab (2013) considered a multi-boundary system and applied the source functions to analyze transient behaviour in horizontal wells. Most models developed have authors considering isotropic cases which is not always the case for many oil reservoirs and even for the anisotropic cases considered, the models presented have not considered the effect of each of the boundaries and all the transition flow periods. We intend to develop a model that accounts for all the aspects of the reservoir with anisotropic permeability and handle each boundary effect separately.

Description of the Physical Model

The physical model used to develop this model is as shown in Figure 1. The drainage volume considered is of length x_e in the x-direction, width y_e in the y-direction and thickness h in the z-direction. We consider a horizontal well of length L which is placed parallel to the x-axis. The well is centrally located at (x_w, y_w, z_w) stretching a length of $L/2$ from x_w in both directions along x.

Mathematical Description

Considering the diffusivity equation (Lee, 1982)

$$\nabla^2 P = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \tag{1}$$

Where P is pressure, c is compressibility, Φ is porosity, μ is fluid viscosity and k is the formation permeability.

Keeping porosity constant and considering a slightly compressible fluid, when we consider permeability anisotropy and ignore gravitational effects, the heterogeneous three dimensional diffusivity equation becomes (Odeh & Babu, 1989)

$$k_x\frac{\partial^2 P}{\partial x^2} + k_y\frac{\partial^2 P}{\partial y^2} + k_z\frac{\partial^2 P}{\partial z^2} = \Phi\mu c_t\frac{\partial P}{\partial t} \tag{2}$$

Where k_x , k_y and k_z are the respective permeabilities in the x, y and z-directions and c_t is the total compressibility.

Dimensionless Pressure

The Dimensionless pressure, P_D is given by (Mathews & Russell, 1967)

$$P_D = \frac{kh\Delta P}{141.2q\mu B} \tag{3}$$

Which can be written as

$$P_D = \frac{2\pi kh}{q\mu}\Delta P \tag{4}$$

Or

$$P_D = 2\pi h_D \int_0^{t_D} s(x_D,\tau_D)\cdot s(y_D,\tau_D)\cdot s(z_D,\tau_D)d\tau_D \tag{5}$$

With;

$$\Delta P = \frac{q}{\Phi c_t L}\int_0^t S(x,y,z,\tau)d\tau \tag{6}$$

Where q is the flow rate, B is the formation volume factor and τ is a dummy variable for time, t.

The product $s(x,y,z,t)$ is obtained using the Newman product rule (Gringarten & Ramey, 1973) and is given by;

$$s(x,y,z,t) = s(x,t) \cdot s(y,t) \cdot s(z,t) \tag{7}$$

Where $s(x,t)$, $s(y,t)$ and $s(z,t)$ are appropriate source and Green's functions.

Source Functions

For a horizontal line source of length L with a centre at (x_w, y_w, z_w) the source function is a result of intersection of three sources. Gringarten and Ramey (1973) presented the source and Green's functions namely;

(a) An infinite plane source at $y = y_w$ in an infinite-slab reservoir of thickness y_e with closed boundaries as shown in Figure 2.

(b) An infinite-slab source of thickness L at $x = x_w$ in an infinite-slab reservoir of thickness x_e with closed boundaries as shown in Figure 3.

(c) An infinite plane-source at $z = z_w$ in an infinite-slab reservoir of thickness h with closed boundaries as shown in Figure 4.

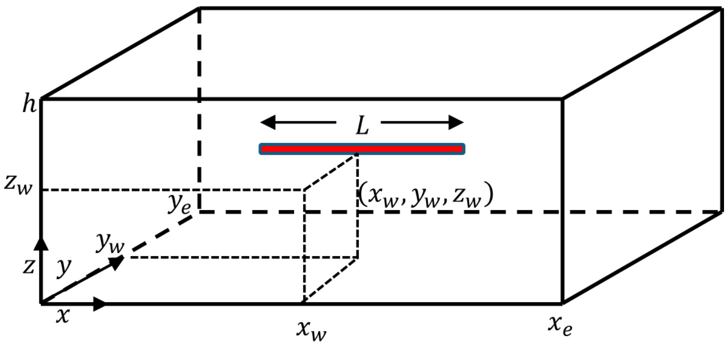

Figure 1: Physical Model of a Horizontal Well in a Rectangular Drainage Volume

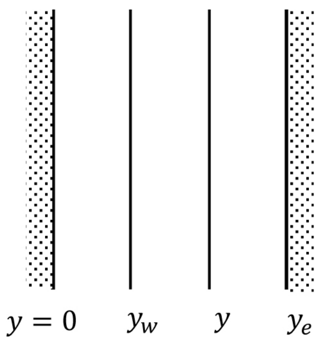

Figure 2: Infinite-plane Source in y-direction

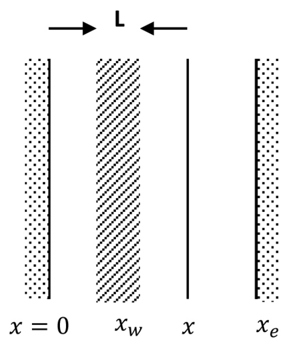

Figure 3: Infinite-slab Source in x-direction

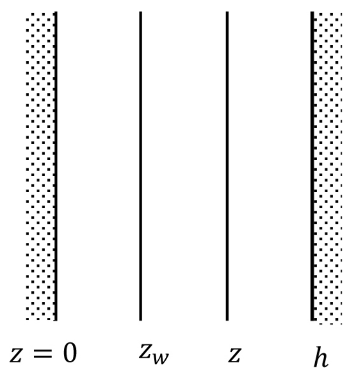

Figure 4: Infinite-plane Source in the z-direction

RESULTS AND DISCUSSION

Considering the drainage system as described in Figure 1, for any given well length and for different oil reservoir parameters, we can separately consider a case where none of the oil reservoir boundaries affect the flow, a case where the vertical boundaries start having an influence on the flow and finally a case where either of the horizontal boundaries have influence on the flow. Considering these cases we present the mathematical models that can be used to approximate pressure for an early time flow period.

Short Time Approximation for Pressure

For short time approximation, where the oil reservoir boundaries have no effect on the flow, we consider, the following source functions (Gringarten & Ramey, 1973)

(a) The approximate form for instantaneous source function during early and intermediate time in the x- direction;

$$s(x,t) = \frac{1}{2}\left[\operatorname{erf}\left(\frac{L/2+(x-x_w)}{2\sqrt{\eta_x t}}\right) + \operatorname{erf}\left(\frac{L/2-(x-x_w)}{2\sqrt{\eta_x t}}\right)\right] \tag{8}$$

(b) The approximate form for instantaneous source function during early times in the y-direction;

$$s(y,t) = \frac{1}{2\sqrt{\pi\eta_y t}}\exp\left[-\frac{(y-y_w)^2}{4\eta_y t}\right] \tag{9}$$

(c) And the approximate form for instantaneous source function during early times in the z-direction;

$$s(z,t) = \frac{1}{2\sqrt{\pi\eta_z t}}\operatorname{erf}\left[-\frac{(z-z_w)^2}{4\eta_z t}\right] \tag{10}$$

Where η_i is the diffusivity constant in the i-axial directions, x, y and z.

Substituting in equation (4), substituting the following dimensionless variables,

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \tag{11}$$

$$i_{wD} = \frac{2i_w}{L}\sqrt{\frac{k}{k_i}}, \tag{12}$$

$$i_{eD} = \frac{2i_e}{L}\sqrt{\frac{k}{k_i}}, \tag{13}$$

Where $i = x, y, z$ and $z_e = h$

$$t_D = \frac{4kt}{\Phi\mu c_t L^2} \tag{14}$$

$$\eta_i = \frac{k_i}{\Phi\mu c_t} \tag{15}$$

Then simplifying, we obtain;

$$P_D = \frac{h_D}{4}\frac{k}{\sqrt{k_y k_z}}\int_0^{t_D}\left\{\left[\operatorname{erf}\left(\frac{\sqrt{k/k_x}+(x_D-x_{wD})}{2\sqrt{\tau_D}}\right) + \operatorname{erf}\left(\frac{\sqrt{k/k_x}-(x_D-x_{wD})}{2\sqrt{\tau_D}}\right)\right]\cdot \left[\exp\left(-\frac{(y_D-y_{wD})^2+(z_D-z_{wD})^2}{4\tau_D}\right)\right]\right\}\frac{d\tau_D}{\tau_D} \tag{16}$$

To be able to analyse the slope at a given pressure point and thus be able to identify the flow regimes, we derive the dimensionless pressure derivative $P_D{}'P_D{}'$ as;

$$P_D{}' = \frac{h_D}{4}\frac{k}{\sqrt{k_y k_z}}\left\{\left[\operatorname{erf}\left(\frac{\sqrt{k/k_x}+(x_D-x_{wD})}{2\sqrt{t_D}}\right)+\operatorname{erf}\left(\frac{\sqrt{k/k_x}-(x_D-x_{wD})}{2\sqrt{t_D}}\right)\right]\cdot \left[\exp\left(-\frac{(y_D-y_{wD})^2+(z_D-z_{wD})^2}{4t_D}\right)\right]\right\} \tag{17}$$

This model presented by equation (16) can be applied in approximating pressure distribution for a horizontal well in a completely bounded oil reservoir for a short time when the flow is not being affected by any of the reservoir boundaries. The model finds application for approximating pressure immediately a well is put into production. Since none of the boundaries of the reservoir have any effect on the flow, the model thus considers a full infinite acting flow period. From the reservoir drainage design, the model shows that the formation thickness of the reservoir as it compares to the anisotropic permeability in both the y and z-directions will influence the pressure distribution. The model can apply for any given well length, for any given reservoir parameters. It's important to note that since the formation thickness is generally small compared to the length of the well, then for high permeability in the y and z-directions, the pressure response will occur within a very short time.

Effect of the Boundaries

When the flow reaches the vertical boundaries, and has not reached the horizontal boundaries, then we consider the following source functions;

(a) The approximate form for instantaneous source function during early and intermediate time in the x- direction defined in equation (8),

(b) The approximate form for instantaneous source function during early times in the y-direction defined in equation (9), and

(c) The instantaneous source function in the z-direction (Gringarten & Ramey, 1973) given by,

$$s(z,t) = \frac{1}{h}\left\{1 + 2\sum_{l=1}^{\infty}\exp\left[-\frac{l^2\pi^2\eta_z t}{h^2}\right]\cos\frac{l\pi z_w}{h}\cos\frac{l\pi z}{h}\right\} \tag{18}$$

Substituting these source functions in equation (4), simplifying and writing in dimensionless form we obtain;

$$P_D = \frac{\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_0^{t_D}\left\{\operatorname{erf}\left[\left(\frac{\sqrt{k/k_x}+(x_D-x_{wD})}{2\sqrt{\tau_D}}\right)\right]+\operatorname{erf}\left[\left(\frac{\sqrt{k/k_x}-(x_D-x_{wD})}{2\sqrt{\tau_D}}\right)\right]\left[\exp\left(-\frac{(y_D-y_{wD})^2}{4\tau_D}\right)\right]\left[1+2\sum_{l=1}^{\infty}\exp\left(-\frac{l^2\pi^2\tau_D}{h_D{}^2}\right)\cos\frac{l\pi z_{wD}}{h_D}\cos\frac{l\pi z_D}{h_D}\right]\right\}\frac{d\tau_D}{\sqrt{\tau_D}} \tag{19}$$

The dimensionless pressure derivative will be;

$$P_D{}' = \frac{\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\left\{\operatorname{erf}\left[\left(\frac{\sqrt{k/k_x}+(x_D-x_{wD})}{2\sqrt{t_D}}\right)\right]+\operatorname{erf}\left[\left(\frac{\sqrt{k/k_x}-(x_D-x_{wD})}{2\sqrt{t_D}}\right)\right]\left[\exp\left(-\frac{(y_D-y_{wD})^2}{4t_D}\right)\right]\left[1+2\sum_{l=1}^{\infty}\exp\left(-\frac{l^2\pi^2 t_D}{h_D{}^2}\right)\cos\frac{l\pi z_{wD}}{h_D}\cos\frac{l\pi z_D}{h_D}\right]\right\}\sqrt{t_D} \tag{20}$$

Equation (19) provides the mathematical model that can be applied to approximate pressure response for a short time from the time the upper and lower boundaries of the oil reservoir start influencing the flow. This model will find application as long as the horizontal boundaries of the oil reservoir do not have any influence on the flow. For this model, the effective permeability relative to the permeability in the y-direction will influence the pressure distribution. Since the flow has already reached the vertical boundary, then width of the reservoir will have an influence on how long this flow period will take. For a reservoir with high horizontal permeability in the y-direction, again the pressure response will take a very short time.

When the flow reaches the parallel horizontal boundary while the perpendicular horizontal boundary and the vertical boundary have not been reached, then we consider;

(a) The approximate form for instantaneous source function during early and intermediate time in the x- direction defined in equation (8),

(b) The instantaneous source function in the y-direction (Gringarten & Ramey, 1973) defined given by,

$$s(y,t) = \frac{1}{y_e}\left\{1 + 2\sum_{m=1}^{\infty} \exp\left[-\frac{m^2\pi^2\eta_y t}{{y_e}^2}\right]\cos\frac{m\pi y_w}{y_e}\cos\frac{m\pi y}{y_e}\right\} \tag{21}$$

(c) And the approximate form for instantaneous source function during early times in the z-direction defined in equation (10).

Substituting in P_D, we obtain;

$$P_D = \frac{\sqrt{\pi}h_D}{2y_{eD}}\sqrt{\frac{k}{k_z}}\int_0^{t_D}\left\{\text{erf}\left[\left(\frac{\sqrt{k/k_x}+(x_D-x_{wD})}{2\sqrt{\tau_D}}\right)\right] + \text{erf}\left[\left(\frac{\sqrt{k/k_x}-(x_D-x_{wD})}{2\sqrt{\tau_D}}\right)\right]\left[1 + 2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2\tau_D}{{y_{eD}}^2}\right)\cos\frac{m\pi y_{wD}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}}\right]\left[\exp\left(-\frac{(z_D-z_{wD})^2}{4\tau_D}\right)\right]\right\}\frac{d\tau_D}{\sqrt{\tau_D}} \tag{22}$$

The dimensionless pressure derivative will be;

$$P_D{}' = \frac{\sqrt{\pi}h_D}{2y_{eD}}\sqrt{\frac{k}{k_z}}\left\{\text{erf}\left[\left(\frac{\sqrt{k/k_x}+(x_D-x_{wD})}{2\sqrt{t_D}}\right)\right] + \text{erf}\left[\left(\frac{\sqrt{k/k_x}-(x_D-x_{wD})}{2\sqrt{t_D}}\right)\right]\left[1 + 2\sum_{m=1}^{\infty} exp\left(-\frac{m^2\pi^2 t_D}{{y_{eD}}^2}\right)\cos\frac{m\pi y_{wD}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}}\right]\left[\exp\left(-\frac{(z_D-z_{wD})^2}{4t_D}\right)\right]\right\}\sqrt{t_D} \tag{23}$$

This model will apply when the y-boundary is influencing the flow but the other boundaries have no effect. The thickness of the oil reservoir relative to the effective permeability and permeability in the z-direction will influence the pressure distribution. With the width of the well-being inversely proportional to the pressure response, then for a small reservoir thickness compared to the length of the well, for reservoirs with high vertical permeability in the z-direction, a short width would increase the pressure response.

When the flow reaches the perpendicular horizontal boundary before the other boundaries are reached, we consider;

(a) The instantaneous source function in the x-direction (Gringarten & Ramey, 1973) and is given by,

$$s(x,t) = \frac{L}{x_e}\left\{1 + \frac{4x_e}{\pi L}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\frac{n^2\pi^2\eta_x t}{{x_e}^2}\right]\sin\frac{n\pi L}{2x_e}\cos\frac{n\pi x_w}{x_e}\cos\frac{n\pi x}{x_e}\right\} \tag{24}$$

(b) The approximate form for instantaneous source function during early time in the y- direction defined in equation (9), and

(c) The approximate form for instantaneous source function during early times in the z-direction defined in equation (10).

Substituting in P_D and simplifying we obtain;

$$P_D = \frac{h_D}{2x_{eD}}\frac{k}{\sqrt{k_y k_z}}\int_0^{t_D}\left\{\left[1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2\tau_D}{x_{eD}{}^2}\right)\sin\frac{n\pi}{x_{eD}}\cos\frac{n\pi x_{wD}}{x_{eD}}\cos\frac{n\pi x_D}{x_{eD}}\right]\cdot\left[\exp\left(-\frac{(y_D-y_{wD})^2}{4\tau_D}\right)\right]\cdot\left[\exp\left(-\frac{(z_D-z_{wD})^2}{4\tau_D}\right)\right]\right\}\frac{d\tau_D}{\tau_D} \quad (25)$$

The dimensionless pressure derivative will be;

$$P_D{}' = \frac{h_D}{2x_{eD}}\frac{k}{\sqrt{k_y k_z}}\left\{\left[1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t_D}{x_{eD}{}^2}\right)\sin\frac{n\pi}{x_{eD}}\cos\frac{n\pi x_{wD}}{x_{eD}}\cos\frac{n\pi x_D}{x_{eD}}\right]\cdot\left[\exp\left(-\frac{(y_D-y_{wD})^2}{4t_D}\right)\right]\cdot\left[\exp\left(-\frac{(z_D-z_{wD})^2}{4t_D}\right)\right]\right\} \quad (26)$$

This model will apply when the x boundary has an effect on the flow while the other boundaries don't. This model shows that the oil reservoir thickness relative to the horizontal permeability in the y-direction, vertical permeability and oil reservoir length will influence the pressure distribution. Depending on the length of the well compared to the length of the oil reservoir, the pressure distribution will be inversely proportional to the reservoir length relative to horizontal permeability in the y-directions and vertical permeability. For high permeability oil reservoirs, a short reservoir length will increase the pressure response.

When the flow has reached both horizontal boundaries but has not reached the vertical boundary, then we consider;

(a) The instantaneous source function in the x-direction defined in equation (24),

(b) The instantaneous source function in the y- direction defined in equation (21), and

(c) The approximate form for instantaneous source function during early times in the z-direction defined in equation (10).

Substituting in P_D and simplifying we obtain;

$$P_D = \frac{\sqrt{\pi}h_D}{x_{eD}y_{eD}}\sqrt{\frac{k}{k_z}}\int_0^{t_D}\left\{\left[1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2\tau_D}{x_{eD}{}^2}\right)\sin\frac{n\pi}{x_{eD}}\cos\frac{n\pi x_{wD}}{x_{eD}}\cos\frac{n\pi x_D}{x_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2\tau_D}{y_{eD}{}^2}\right)\cos\frac{m\pi y_{wD}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}}\right]\cdot\left[\exp\left(-\frac{(z_D-z_{wD})^2}{4\tau_D}\right)\right]\right\}\frac{d\tau_D}{\sqrt{\tau_D}} \quad (27)$$

The dimensionless pressure derivative will be;

$$P_D{}' = \frac{\sqrt{\pi}h_D}{x_{eD}y_{eD}}\sqrt{\frac{k}{k_z}}\left\{\left[1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t_D}{x_{eD}{}^2}\right)\sin\frac{n\pi}{x_{eD}}\cos\frac{n\pi x_{wD}}{x_{eD}}\cos\frac{n\pi x_D}{x_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2 t_D}{y_{eD}{}^2}\right)\cos\frac{m\pi y_{wD}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}}\right]\cdot\left[\exp\left(-\frac{(z_D-z_{wD})^2}{4t_D}\right)\right]\right\}\sqrt{t_D} \quad (28)$$

This model will apply in pressure approximation when both the x and y-boundaries have an effect on the flow but the z –axis does not. This model shows an inverse proportionality between the pressure distribution and both the width and length of the reservoir relative to the permeability in the z-directions. For on a small oil reservoir thickness compared to the length of the well, pressure response will increase, when the length and width of the oil reservoir are short for high vertical permeability.

When the flow reaches the horizontal boundaries perpendicular to the well and the vertical boundary but has not reached the horizontal boundary parallel to the well, we consider;

(a) The instantaneous source function in the x-direction defined by equation (24),

(b) The approximate form for instantaneous source function during early times in the y-direction defined in equation (9), and

(c) The instantaneous source function in the z-direction defined in equation (18).

Substituting and simplifying in a similar manner as described earlier, we obtain;

$$P_D = \frac{\sqrt{\pi}}{x_{eD}} \sqrt{\frac{k}{k_y}} \int_0^{t_D} \left\{ \left[1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} exp\left(-\frac{n^2\pi^2\tau_D}{{x_{eD}}^2} \right) \sin\frac{n\pi}{x_{eD}} \cos\frac{n\pi x_{wD}}{x_{eD}} \cos\frac{n\pi x_D}{x_{eD}} \right] \left[\exp\left(-\frac{(y_D - y_{wD})^2}{4\tau_D} \right) \right] \left[1 + 2\sum_{l=1}^{\infty} exp\left(-\frac{l^2\pi^2\tau_D}{{h_D}^2} \right) \cos\frac{l\pi z_{wD}}{h_D} \cos\frac{l\pi z_D}{h_D} \right] \right\} \frac{d\tau_D}{\sqrt{\tau_D}} \quad (29)$$

The dimensionless pressure derivative of equation (29) is given by;

$$P_D{}' = \frac{\sqrt{\pi}}{x_{eD}} \sqrt{\frac{k}{k_y}} \left\{ \left[1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} exp\left(-\frac{n^2\pi^2 t_D}{{x_{eD}}^2} \right) \sin\frac{n\pi}{x_{eD}} \cos\frac{n\pi x_{wD}}{x_{eD}} \cos\frac{n\pi x_D}{x_{eD}} \right] \left[\exp\left(-\frac{(y_D - y_{wD})^2}{4t_D} \right) \right] \left[1 + 2\sum_{l=1}^{\infty} exp\left(-\frac{l^2\pi^2 t_D}{{h_D}^2} \right) \cos\frac{l\pi z_{wD}}{h_D} \cos\frac{l\pi z_D}{h_D} \right] \right\} \sqrt{t_D} \quad (30)$$

The model described in equation (29) can be applied to approximate the pressure distribution when both the z and x boundaries are influencing the flow but the y boundary is yet to affect the flow. The length of the oil reservoir in this model relative to the horizontal permeability in the y- direction will be inversely proportional to the pressure distribution. The pressure response will increase if the length of the reservoir is small in a case where the oil reservoir has high horizontal permeability in the y-direction.

On the other hand when the flow reaches the horizontal boundaries parallel to the well and the vertical boundaries before reaching the horizontal boundary perpendicular to the well, we consider;

(a) The approximate form for instantaneous source function during early and intermediate time in the x- direction defined in equation (8),

(b) The instantaneous source function in the y-direction defined by equation (21), and

(c) The instantaneous source function in the z-direction defined in equation (18).

Substituting and simplifying as described earlier, we obtain;

$$P_D = \frac{\pi}{y_{eD}} \int_0^{t_D} \left\{ \left[\text{erf}\left(\frac{\sqrt{k/k_x} + (x_D - x_{wD})}{2\sqrt{\tau_D}} \right) + \text{erf}\left(\frac{\sqrt{k/k_x} + (x_D - x_{wD})}{2\sqrt{\tau_D}} \right) \right] \cdot \left[1 + 2\sum_{m=1}^{\infty} exp\left(-\frac{m^2\pi^2\tau_D}{{y_{eD}}^2} \right) \cos\frac{m\pi y_{wD}}{y_{eD}} \cos\frac{m\pi y_D}{y_{eD}} \right] \cdot \left[1 + 2\sum_{l=1}^{\infty} exp\left(-\frac{l^2\pi^2\tau_D}{{h_D}^2} \right) \cos\frac{l\pi z_{wD}}{h_D} \cos\frac{l\pi z_D}{h_D} \right] \right\} d\tau_D \quad (31)$$

The dimensionless pressure derivative is given by;

$$P_D{}' = \frac{\pi}{2y_{eD}} \sqrt{\frac{k}{k_y}} \left\{ \left[\text{erf}\left(\frac{\sqrt{k/k_x} + (x_D - x_{wD})}{2\sqrt{t_D}} \right) + \text{erf}\left(\frac{\sqrt{k/k_x} + (x_D - x_{wD})}{2\sqrt{t_D}} \right) \right] \cdot \left[1 + 2\sum_{m=1}^{\infty} exp\left(-\frac{m^2\pi^2 t_D}{{y_{eD}}^2} \right) \cos\frac{m\pi y_{wD}}{y_{eD}} \cos\frac{m\pi y_D}{y_{eD}} \right] \cdot \left[1 + 2\sum_{l=1}^{\infty} exp\left(-\frac{l^2\pi^2 t_D}{{h_D}^2} \right) \cos\frac{l\pi z_{wD}}{h_D} \cos\frac{l\pi z_D}{h_D} \right] \right\} t_D \quad (32)$$

The model described in equation (31) can be applied to approximate the pressure distribution when the y and z-boundaries start influencing the flow before the x boundaries start having an effect on the flow. For this model it's the oil reservoir's width that will be inversely proportional to the pressure distribution. Thus, a short reservoir width will increase the pressure response.

CONCLUSION

From the models derived, it is possible to;

(a) Determine possible flow periods,

(b) Suggest possible transitional flow periods,

(c) Determine reservoir geometry, and

(d) Determine the reservoir permeability and permeability anisotropy.

The reservoir geometry and directional permeability will affect the number and duration of flow periods. Considering the vertical and horizontal permeability as well as the length of the well compared to both the length, width and thickness of the reservoir, the flow periods can be determined and the transitional periods suggested. This will affect the performance of the well. Thus, depending on the location of the reservoir, the viability of the reservoir will be determined when these parameters are carefully considered and substituted in the models.

Nomenclature

B	Formation volume factor, rbbl/stb
c_t	Total compressibility, 1/psi
h	Reservoir thickness, ft
h_D	Dimensionless reservoir thickness
i	Axial flow direction; x, y and z
k	Reservoir permeability, md
k_x	Permeability in the x-direction, md
k_y	Permeability in the y-direction, md
k_z	Permeability in the z-direction, md
l	Well length, ft
P_D	Dimensionless pressure
$P_D{}' = t_D \frac{\partial P_D}{\partial t_D}$	Dimensionless pressure derivative
q	Flow rate, bbl/day
s	Source
t	Time, hours
t_D	Dimensionless time
x	Length in x-direction, ft
x_D	Dimensionless reservoir length
x_e	Reservoir length, ft
x_{eD}	Dimensionless reservoir length
x_w	Source coordinate in the x-direction, ft
x_{wD}	Dimensionless source coordinate in the x-direction
y	Width in y-direction, ft
y_D	Dimensionless reservoir width
y_e	Reservoir width, ft
y_{eD}	Dimensionless reservoir width
y_w	Source coordinate in the y-direction, ft
y_{wD}	Dimensionless source coordinate in the y-direction
z	Thickness in z-direction, ft
z_D	Dimensionless reservoir thickness
z_w	Source coordinate in the z-direction, ft
z_{wD}	Dimensionless source coordinate in the z-direction
η_i	Diffusivity constant in the i axial direction, md-psi/cp
Φ	Porosity, fraction
μ	Resrvoir fluid viscosity, cp
τ_D	Dimesnsionless dummy variable for time

Superscripts

$'$	Derivative

Subscripts

D	Dimensionless
e	External
i	Axial direction
w	Wellbore

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